Supplies

Sheet of 1" grid paper. Small counters such as beans, buttons, beads, or counting cubes, cards for the numbers 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, and 20. Graph paper and markers.

The Activity

The student will choose a number card and lay out that number of counters in equal rows and columns. For instance, 18 could be laid out in 3 columns of 6 (6 groups of 3) or 2 columns of 9 (9 groups of 2). The student will discuss with instructor how many equal groups can be formed from 18. The student records this as division sentences. Then the instructor rotates the mat so that the reverse shows, namely 6 columns of 3 (3 groups of 6) or 9 columns of 2 (2 groups of 9). The student will record the new division sentences, while identifying if the division sentences are different or the same.

Variations

• The student can make equal groups by dividing the objects into sorting bowls or paper plates.

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Then explain what you will do. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will pick a product number and lay out the arrays for that number and record the math sentences.

Questions: How many rows/groups are in your array? How many are in each group? How can you use the same numbers for a different division sentence? How many groups of 5 are in twenty?

Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? Which was the largest column? Which one the smallest? Why do you think that is?

Math Observation Checklist:

This activity will give insight into the student's understanding of order of numbers, understanding of division, attending to more than one piece of information, and attending to relevant information.
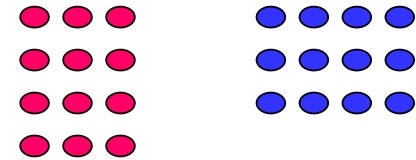
12 is 4 groups of 3

12 is 3 groups of 4

$$12 \div 4 = 3$$

$$12 \div 3 = 4$$

MILE
Math Interactive
Learning Experience

Supplies

Wide wooden popsicle sticks, small stickers. Cards with division instructions such as "share 12 among 6 groups (popsicle sticks)". Small counters such as beans or buttons. Paper plates. Graph paper and markers.

The Activity

The student will choose a division instruction card and follow the instruction by choosing the appropriate number of popsicle sticks (groups) and count out the required number of stickers. The student puts the required number of stickers on each popsicle stick. The student records the division sentences and chooses another instruction card.

Variations

- The student can make equal groups by dividing the counters into sorting bowls or paper plates and recording the division sentence.

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Then explain what you will do. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will pick an instruction card and decides how many popsicle sticks/paper plates are needed and how many stickers/counters.

Questions: How many popsicle sticks do you need? How many stickers? How do you know that? How will you divide up your stickers among the popsicle sticks?

Reflect:

During and after the activity reflect on what the student is doing/has done.

Questions: What did you do? You divided 12 stickers among 2 sticks and then among 3 sticks. Where were the popsicle sticks with the most stickers? How come?
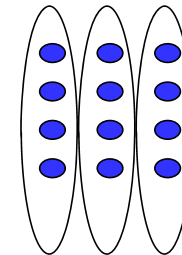
Math Observation Checklist:

This activity will give insight into the student's understanding of division, attending to more than one piece of information, and attending to relevant information.

Share 12 among 2 groups

Share 12 among 3 groups

$12 \; / . \; 2 = 6$

$12 \; / . \; 3 = 4$

![MILE - Math Interactive Learning Experience]
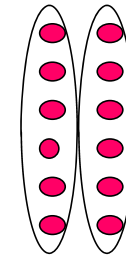
Supplies

Cards with remainder-division instructions such as "share 14 among 6 groups." Small counters such as beans or buttons. Paper plates or cups. Graph paper and markers.

The Activity

The student will choose a remainder division instruction card and follow the instruction by choosing the appropriate number of cups (groups) and counting out the required number of counters. Then the student puts a counter in each cup until there are not enough counters to put in each cup. These remaining counters are "leftovers." The student records the division sentences. For instance, 14 counters shared among 6 groups is 2 in each group and 2 leftover.

Variations

- The student can make a table out of the math facts for the division sentences.

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Then explain what you will do. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will pick an instruction card and decides how many paper plates/cups are needed and how many counters. Then they will divide the counters.

Questions: How many cups do you need? How many counters? How do you know that? How will you divide up your counters among the cups?

Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? You divided 15 counters among 2 cups. What happened?

Math Observation Checklist:

This activity will give insight into the student's understanding of division, attending to more than one piece of information, and attending to relevant information.

Share 15 among 2 groups

Share 14 among 3 groups

$$15 \div 2 = 7$$
1 leftover

$$14 \div 3 = 4$$
2 leftover

![MILE - Math Interactive Learning Experience]

Supplies

Sheet of 1" grid paper; cubes that interlock on all sides, or tile counters.

The Activity

The student will work on odd and even numbers. Ask the student to build patterns with the cubes or tiles starting with 2, then 4, 6, and so on. Start the pattern with 2 cubes or tiles in a horizontal position, then build 2 on top for each subsequent number (see model on the right.). Show the student that each tower can be **evenly** split in 2, and discuss that this means it is an **"even"** number. Next, ask student to build patterns for the numbers 1, 3, 5, and so on, keeping the earlier built patterns in tact. Use a different color for the **"odd one out"** (see model on the right.) Discuss that these are **"odd"** numbers. When you split odd numbers in 2 there always is a remainder of 1. Note: help the student understand that all numbers that end with 2, 4, 6, 8 or 0 are always even, no matter how large the number is. Therefore, all numbers that **do not** end in 2, 4, 6, 8 or 0 are odd. **(No need to remember those numbers!)**

Variations

- Ask student to take a handful of cubes or counters, lay them out in an array of 2 columns and determine if the number is even or odd.

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will lay out the patterns for the even and odd numbers.

Questions: If you start with 2 cubes next to each other, how can you make it into a tower of 4? How many blocks did you add? If you make another tower that has 2 more cubes than this one, how many cubes do you have? What happens when you put an extra cube on top of this tower? How can you skip-count the even numbers?

Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? How can you tell a number is "Even"? "Odd"? How can you tell if a really high number is even or odd?
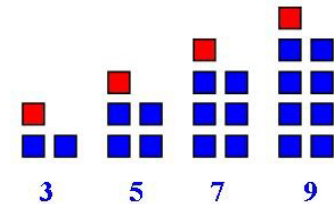
Math Observation Checklist:

This activity will give insight into the student's ability to understand odd and even numbers; division; attend to more than one piece of information, and attend to relevant information.

Even numbers can be split evenly into 2 groups

2   4   6   8
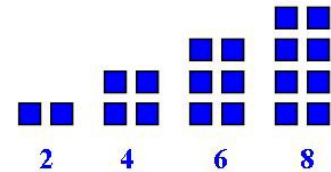
Odd numbers have an Odd one out

3   5   7   9

MILE
Math Interactive
Learning Experience

Supplies

Sheet of 1" grid paper; Base ten rods and cubes (for tens and ones)

The Activity

The student will work on long division without remainder or regrouping. You will need to help the student understand how to write the division symbols and where to put the answers. With the manipulatives the student can "see" the division. **Division is different from other math operations, in that it starts on the left side.** When the student uses the base 10 rods and cubes, make sure that the sequence is correct and that they starts by dividing the tens-column. The example on the right shows how you can discuss the sequence with the student.

Variations

*   Ask student to divide a 3-digit number. Make sure the hundreds-column has a number that can be divided by the number you chose, so that no regrouping into the tens column needs to take place.

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Then explain what you will do. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will use the manipulatives to perform the division and record the answer in a long-division format.

Questions: Where do you need to start when you divide 46 by 2? Can you show me how you do that? In what column are you putting the answer?

Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? Did division remind you of something? (multiplication).

Math Observation Checklist:

This activity will give insight into the student's ability to understand whole numbers; divide numbers into even groups; attend to more than one piece of information, and attend to relevant information.

Tens	Ones
1st	2nd

$46 \div 2 = ?$

$$\begin{array}{r} 23 \\ 2\overline{)46} \\ 4 \\ \hline 06 \\ 6 \\ \hline 0 \end{array}$$
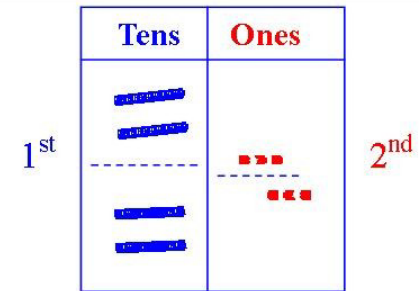
1. **Divide the tens by 2**
    Write the 2 above the 4 (the tens-column)
    Multiply 2 x 2 and write 4.
    Subtract
2. **Bring down 6 and divide by 2.**
    Write the 3 above the 6
    Multiply 3x2 and write 6.
    Subtract. Done!

![MILE logo] MILE — Math Interactive Learning Experience

Supplies

Sheet of 1" grid paper; Base ten flats, rods and cubes (for tens and ones)

The Activity

The student will work on long division with regrouping. You will need to help the student understand how to write the division symbols and where to put the answers. With the manipulatives the student can "see" the division. Division is different from other math operations, in that it starts on the left side. When the student uses the base 10 rods and cubes, make sure that the sequence is correct and that they start by dividing the tens-column. Once the tens-column has been divided, there will be leftover tens. These need to be put into the ones-column and then the ones-column can be divided. The example on the right shows how you can discuss the sequence with the student .

Variations

- The student can make pieces of "fruit" with the play-doh according to the number on the number/dot card.
- The student can line up the sets in the same pattern as the dot cards.
- The student creates dot and/or numeral cards.

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Then explain what you will do. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will use the manipulatives to perform the division and record the answer in a long-division format.

Questions: Where do you need to start when you divide 56 by 2? Can you show me how you do that? In what column are you putting the answer?

Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? Did division remind you of something? (multiplication).

Math Observation Checklist:

This activity will give insight into the student's ability to understand whole numbers; understand that division is the opposite of multiplication; long division with regrouping; attend to more than one piece of information, and attend to relevant information.

Tens	Ones

$56 \div 2 = ?$

$$2\overline{)56}$$
28

1. **Divide the tens by 2**
   Write the 2 above the 5 (the tens-column)
   Multiply 2 x 2 and write 4.
   Subtract: remainder 1 ten
2. **Bring down 6 and divide 16 by 2.**
   Write the 8 above the 6
   Multiply 8x2 and write 16.
   Subtract. **Done!**

MILE
Math Interactive
Learning Experience

Supplies

Sheet of 1" grid paper; Counting tiles; Markers or pencils.

The Activity

The student will evenly divide a number by as many different numbers as possible ("factors".) Explain that some numbers can only be evenly divided by 1 or by itself.

These are called "prime numbers", for instance 2, 3, 7 and 11.

The rules for dividing are as follows:
a number can be divided by:
2 if the last digit is 0, or even
3 if the **sum** of all digits can be divided by 3
4 if the last 2 digits can be divided by 4
5 if the last digit is a 5 or a 0
10 if the last digit is 0
With your student, decide on a number to be factored. Ask the student to use the counters to lay out the different groups that the number can be divided into. Then, the number of groups and the number of counters in that group will be the factors.

Variations

• Ask student to find the factors for two numbers and determine the **Greatest Common Factor (GCF)**. The GCF is the largest factor that the 2 numbers have in common.

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Then explain what you will do. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will use the manipulatives to find the factors and record the answers.

Questions: What number do you always start with when you want to find the factors (1)? So, what is the first factor for 56? Can you show me how you write that down?

Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? How many factors did you find for 24? When you had a number, how did you find the factors? (Answer: by division) How can you find the number when you know the 2 factors? (Answer: by multiplying). What did you find out about multiplying and dividing?

Math Observation Checklist:

This activity will give insight into the student's ability to understand division is the opposite of multiplication; factoring; division; attend to more than one piece of information, and attend to relevant information.

What are the factors for 18?
(How many different ways can we divide 18?)

Can we divide 18 by 1? **Yes!** We get 18.
$18 \div 1 = 1$ group of 18. Factors: **1** and **18**

Can we divide 18 by 2? **Yes!** We get 9
$18 \div 2 = 2$ groups of 9. Factors: **2** and **9**

Can we divide 18 by 3? **Yes!** We get 6
$18 \div 3 = 3$ groups of 6. Factors: **3** and **6**

Can we divide 18 by 4? **No!** (Try it out!)

Can we divide 18 by 5? **No!** (Try it out!)

So, the factors for 18 are:
1, 2, 3, 6, 9, 18

Trick! When we have 3 groups of 6, we can "flip" and have 6 groups of 3!
So, we don't need to go further than trying out 5!

MILE
Math Interactive
Learning Experience

Supplies

Sheet of 1" grid paper; Counting tiles; Markers or pencils

The Activity

The student will find the **prime factors** for a number. Prime factors are the numbers that cannot be divided by any number, except itself. For instance 3, 5 or 13. When factoring, when you reach a "prime number" such as 3 or 5, you cannot continue to divide. To find the prime factors, start by dividing the number you chose (for instance 18) by the smallest prime number (that is 2). The student will write 18 on the paper with 2 arrows pointing down. 18 can be divided into 2 groups of 9 and the student will lay out the two groups. They will then write 2 and 9 under the arrows pointing down from 18, and circle the 2.

Next, one group of 9 will be removed and the student will determine if the remaining group of 9 canbe further divided.

Nine cannot be divided by 2, so we try the next prime number, which is 3. Nine can be divided into 3 groups of 3, and the student lays out the groups. They draw 2 arrows down from the 9 and write 3 and 3, circling the first 3.

Two groups of 3 are removed and the student determines if the remaining group of 3 can be further divided. No. So the last 3 is circled. The circled numbers (2, 3, and 3) are the prime factors for 18.

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Then explain what you will do. Formulate a plan with the student.

Questions: What is the plan? What will we do first? Next? And then?

Act:

The student will use the manipulatives to find the prime factors and record the answers.

Questions: What number do you need to start with, when you want to find the prime factors? Why is that? So, what is the first factor for 18? Can you show me how you write that down?
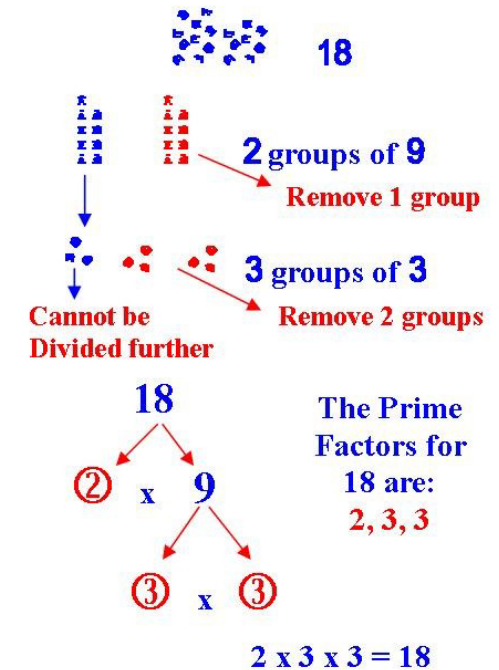
Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? What were the prime factors for the number you chose? When you had a number, how did you find the factors? How can you check if your prime factors are correct?

Math Observation Checklist:

This activity will give insight into the student's ability to understand division is the opposite of multiplication; divide a number among equal groups; factoring; division; attend to more than one piece of information, and attend to relevant information.

18

2 groups of **9**
Remove 1 group

3 groups of **3**
Remove 2 groups

Cannot be Divided further

18

② x **9**

The Prime Factors for 18 are: 2, 3, 3

③ x ③

2 x 3 x 3 = 18

MILE
Math Interactive
Learning Experience

Classroom Activities

Supplies

Sheet of 1" grid paper; small counters such as beans, buttons, beads, or counting cubes; number card with products (i.e. numbers that are answers to multiplication questions, for instance 20, which is 2x10, or 4x5); graph paper and markers.

The Activity

The student will choose a number card and lay out that number of counters in equal rows and columns. For instance 18 could be laid out in 3 columns of 6, or 2 columns of 9. The student will record the multiplication sentences: 3x6=18 and 2x9=18. Then the instructor rotates the mat so that the reverse shows, namely 6 columns of 3 or 9 columns of 2. The student will record the new multiplication sentences, while relating them to the old ones.

Variations

- Have the student roll a number die two times. The 2 numbers represent the multiplication sentence, and the student will lay out the array for those numbers.

Focus:

Encourage the student to focus they attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will pick a product number, lay out the arrays for that number, and record the math sentences.

Questions: How many rows/groups are in your array? How many are in each group? How could we say 5+5+5+5+5 a little shorter? What do you think multiplication is? Trick question! Is 3x5 more or less than 5x3?

Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? What happened when you had 3 groups of 5 and we rotated the mat? What is easier to say, the addition sentence for the groups or the multiplication sentence?

Math Observation Checklist:

This activity will give insight into the student's understanding of order of numbers, understanding of multiplication, attending to more than one piece of information, and attending to relevant information.
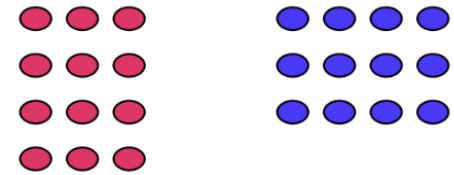
$3 + 3 + 3 + 3 = 12$

$4 + 4 + 4 \quad = 12$

4 groups of 3 are 12

3 groups of 4 are 12

$4 \times 3 = 12$

$3 \times 4 = 12$

MILE
Math Interactive
Learning Experience

Supplies

Sheet of 1" grid paper; multiplication chart; tile counters in different colors; markers; pencils

The Activity

The student will build arrays of numbers by laying out rows and columns of tiles and determine the number sentences for each row. You will introduce the term "factor". For instance, in 3 x 4 = 12, the factors are 3 and 4. Show that the 3 and the 4 can be flipped around (commuted) and the answer will still be 12. Choose a times-table, for instance 3 or 4, and ask the student to lay out the complete table. Note: Multiplication tables are learned in different formats, namely: 1 x 3, 2 x 3, 3 x 3, 4 x 3, and so on; OR 3 x 1, 3 x 2, 3 x 3, 3 x 4, and so on. Check with the student how he/she is learning the tables in school and use that format.

Variations

- Do this exercise with "doubles" such as 2x2, 3x3 and so on. Point out the "square" formation of the tiles laid out, and ask the student to plot the patterns on multiplication chart. (The pattern of the squared pairs will form a diagonal on the chart.)

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will pick a times-table, lay out the arrays for that number, and record the math sentences.

Questions: How many rows are in your array? How many are in each row? When looking at your array, how can you tell which product will be higher, without counting? What makes you say that? How does adding one row change the total? If you know that 3x 4 = 12, what can you tell me about 4 x 3? How can you show that in the array you just made? (Rotate the array.) Tell me what the factors are for the rows you just put down.
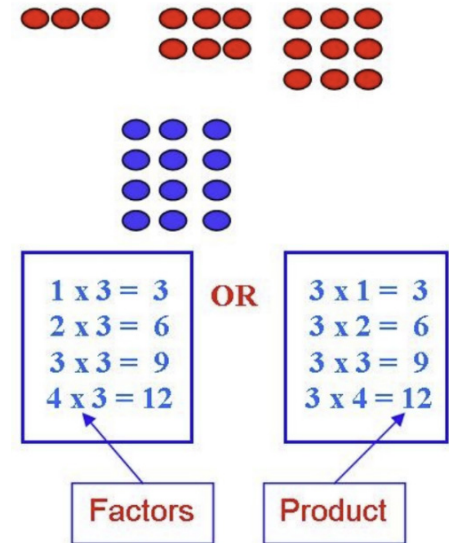
Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? What happened when you had 3 rows of 5 and then added a row? When you had 3x4=12, what are the 3 and 4 called? The 12?

Math Observation Checklist:

This activity will give insight into the student's understanding of multiplication, knowledge of multiplication tables, systematic exploration, attending to more than one piece of information, and attending to relevant information.

1 x 3 = 3	OR	3 x 1 = 3
2 x 3 = 6		3 x 2 = 6
3 x 3 = 9		3 x 3 = 9
4 x 3 = 12		3 x 4 = 12

Factors   Product

![MILE — Math Interactive Learning Experience logo]

Multiplication
Page: 55

Supplies

Sheet of 1" grid paper; multiplication chart; tile counters in different colors;

The Activity

The student will build arrays of numbers by laying out rows and columns of tiles. Explain "rows" and "columns" to the student. The number of rows will increase by one for each array, for instance, one row of 3 tiles, then next to it, 2 rows of 3 tiles, then 3 rows of 3 tiles, and so on. Then the student will determine the number sentences for each row.

Note: Multiplication tables are learned in different formats, namely: 1 x 3, 2 x 3, 3 x 3, 4 x 3, and so on; OR 3 x 1, 3 x 2, 3 x 3, 3 x 4, and soon. Check with the student how they is learning the tables in school and use that format.

Variations

- Do this exercise with different numbers and ask the student to plot the patterns on multiplication chart.

Focus:

Encourage the student to focus their attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will pick a product number, lay out the arrays for that number, and record the math sentences.

Questions: How many rows are in your array? How many are in each row? When looking at your array, how can you tell which product will be higher, without counting? What makes you say that? How does adding one row change the total? If you know that 3 x 4 = 12, what can you tell me about 4 x 3? How can you show that in the array you just made? (Rotate the array.)
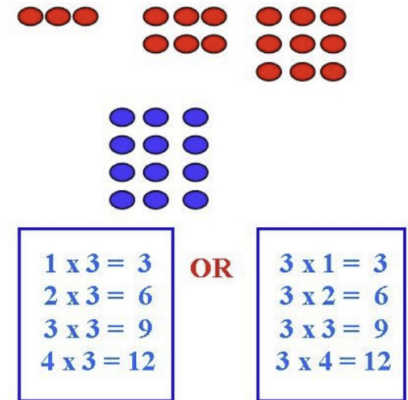
Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? What did you find out about the objects you measured? If something is shorter than something else, can it at the same time be taller than something else? How come?

Math Observation Checklist:

This activity will give insight into the student's understanding of multiplication, knowledge of multiplication tables, systematic exploration, attending to more than one piece of information, and attending to relevant information.

![Array diagram of red and blue tiles with number sentences]

1 x 3 = 3		3 x 1 = 3
2 x 3 = 6	OR	3 x 2 = 6
3 x 3 = 9		3 x 3 = 9
4 x 3 = 12		3 x 4 = 12

Classroom Activities

Supplies

Sheet of 1" grid paper; multiplication chart; Base 10 blocks (cubes, rods, flats, blocks); tens-and-ones chart; number cards or number dice to create the numbers to be multiplied; markers.

The Activity

The student will multiply double-digit numbers by a double-digit number, with regrouping. Ask the student to write down the multiplication problem vertically. Explain that multiplication always starts with the ones column and then the tens column. Discuss for each problem which number is in the ones column and which one is in the tens column. When multiplying with the number in the tens column, it is necessary to "scoot over" by placing a 0 in the ones column.

Use the base 10 blocks to lay out the arrays for the numbers to be multiplied

Variations

- Use three-digit numbers to be multiplied by a two-digit number. In this case a hundreds-tens-ones chart is also needed.

Focus:

Encourage the student to focus they attention on the task at hand. Allow the student to get acquainted with the supplies by touching, holding, and talking about them. Formulate a plan with the student.

Questions: What is the plan? What do you need to do first? Next?

Act:

The student will pick a double digit number, write down the math problem vertically, and lay out the arrays for the ones and the tens for each number to be multiplied, Then they will record the math problem with the product.

Questions: What number needs to be multiplied first? Why do you think that is so? How can you check if you got the right number?

Reflect:

During and after the activity, reflect on what the student is doing/has done.

Questions: What did you do? When you multiplied 7 and 2 what was the product? How could you tell? Why did you need to "scoot " and write a 0 in the ones place when you multiplied with the tens number?

Math Observation Checklist:

This activity will give insight into the student's ability to understand that multiplication is repeated addition, knowledge of multiplication tables, procedure for multiple digit multiplication, systematic exploration, attend to more than one piece of information, and attend to relevant information.